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The Study of Game Theory Using Operation Research

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Abstract: *In this paper, we have developed a new approach to quadratic fractional programming (QFP) problems where high-level objective quadratic and low-level activity is directly separated. In this way the QFP problem is converted into a single-level quadratic programming (QP) problem with some problems by forcing a low-level problem-solving gap to zero. Then by finding all regional vertices with both low-level problems, namely convex polyhedron, one QP level problem is converted into a series of complete QP problems with specific barriers that can be solved with any standard QP solution. Foremost among the best solutions is providing the right solution for the real problem of bi-level programming (BLP). Theoretical results are illustrated with the help of a numerical model.*

Keywords: *Game Theory, Applications, Linear programming, International Relations, Philosophy.*

1. Introduction

Operations Research, also called Decision Science or Operations Analysis, is the study of applying mathematics to business questions [1]. As a sub-field of Applied Mathematics, it has a very interesting position alongside other fields as Data Science and Machine Learning. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis. As a sub-field of Applied Mathematics, it has a very interesting position alongside other fields as Data Science and Machine Learning. Operations Research uses mathematics and statistics to answer optimization and simulation questions. Whenever we translate a business question in an optimization question, it is primordial that we have clear definitions of a cost to minimize or a benefit to maximize.

1.1 Three key items of Operations Research

(1) Algorithms and Statistics:
Operations Research is going to rely heavily on algorithms, mathematics and statistics. A very important family of algorithms in Operations Research are:

Optimization Algorithms: algorithms that try to find a maximum or a minimum, given a certain set of possibilities. As an example of this, we could use an optimization algorithm to minimize the cost of staffing a factory, given a set of constraints on the number of people needed, and constraints of each of the individual employees [2].

(2) Optimization:
Finding the best possible solution to a question, given potential practical constraints. Optimization can be about maximization or Minimization of a cost or benefit that is decided on before starting. It is possible to have multiple goals, in which case we can define a combined cost function by applying weights of our different costs (for example taking the sum of two costs to minimize could be an example of a combined cost function).A second often-occurring thing to deal with in those optimizations are constraints. Sometimes, an algorithm looking for minimization of a cost can go look for solutions in a way that is practically impossible. For example, when looking for the best staff planning, we want to constrain the algorithm to plan people for 24-hour-shifts because that would be simple illegal [3].

(3) Simulation:

Simulation is actually a comparable task to optimization. Rather than asking an algorithm what the best staff planning is, we could also ask an algorithm what the effect of changing the planning would be. This type of task is close to optimization since we could simply use the optimization algorithm with a different input configuration to simulate what would be the optimal outcome with those different inputs.

So in short, Operations Research is applying mathematics to business questions with a goal of optimization and/or simulation.

2. Linear fractional programming (LFP)

Linear fractional programming (LFP) problem is one whose objective function has a numerator and a denominator and are very useful in production planning, financial and corporate planning, health care and hospital planning [4].

General Linear Programming Problem:- A linear programming problem is defined as follows:

$$max\ or\ min\ z = \boldsymbol{cx}$$

subject to the constraints:

$$\boldsymbol{Ax} \geq or \leq or = \boldsymbol{b}$$
$$\boldsymbol{x} \geq 0$$

Where (i) $\boldsymbol{A}$ is an $m \times n$ matrix

(ii) $\boldsymbol{x}$ is $n \times 1$ & $\boldsymbol{b}$ is $m \times 1$ column vector

(iii) $\boldsymbol{c}$ is $1 \times n$ row vector.

Linear Programming:

Linear Programming Problem (LPP) is optimization (maximization or minimization) method build up in the field of Operations Research. LPP have three bits:

(i) Linear function known as objective function,
(ii) Subject to constraints, which is a set of linear inequalities/equalities.
(iii) Non-negative decision variables.

The objective function may be cost, profit, production capacity etc. which is to be obtained in the best optimal (maximize or minimize) manner [4]. The constraints may be imposed by various resources such as availability of raw material, market demand, storage capacity, production process and equipment etc.

Definition 1: A mathematical formulation of linear programming problem is:

$$max. Z\ or\ min. Z = C^T X$$

Subject to:

$$AX \leq \text{ or } = \text{ or } \geq b$$
$$X \geq 0$$

Where,

(i) A is an $m \times n$ matrix,
(ii) b is an $m \times 1$ vector
(iii) X are $n \times 1$ vectors and
(iv) C^T is $1 \times n$ vector.

Example: (Manufacturing problem) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

Formulation:

$$Maximise\ Z = 8000\,x + 12000\,y$$

subject to the constraints:

$9x + 12y \leq 180$ (Fabricating constraint)

i.e. $3x + 4y \leq 60$

$x + 3y \leq 30$ (Finishing constraint)

$x \leq 0,\ y \leq 0$ (Non-negative constraint)

Definition 2: The non negative variables which are subtracted from the LHS to the constraints ($\geq$ type) to convert them into equalities are called the **surplus variables**.

Example:

Following constraint are in the form of inequality

$$x_1 + 5x_2 \geq 21, \quad 2x_1 + 5x_2 \geq 10$$

Convert them into equality as

$$x_1 + 5x_2 - S_1 = 21, \quad 2x_1 + 5x_2 - S_2 = 23$$

Here, S_1 and S_2 are non negative surplus variables.

Definition 3: The non negative variables which are added to the LHS to the constraints ($\leq$ type) to convert them into equalities are called the **slack variables**.

Example: Following constraints are in the form of inequality

$$x_1 - 5x_2 \leq 4, \quad 2x_1 + 3x_2 \leq 5$$

Convert them into equality as

$$x_1 - 5x_2 + S_1 = 4, \quad 2x_1 + 3x_2 + S_2 = 5$$

Where, S_1 and S_2 are non negative slack variables.

Definition 4: In LPP some constraints may have the signs $\geq$ or $=$ In such type of problems we introduce surplus variables in the constraints with sign $\geq$. We cannot get the starting basic matrix $B = I_m$ in these problems. Therefore we add one more variable in each of such constraints to avoid this difficulty. These variables are called **'artificial variables' [5]**.

Standard form of the Linear Programming Problem:

Step1: All the inequality constraints are converted to equalities by introducing the non-negative slack/surplus variables. The value of coefficients of slack/surplus variables in the objective function are always taken zero.

Step2: The right hand side constants of each constraint should be non- negative, if not, then multiply by -1 to the corresponding inequality of the constraints.

Step3: Each variable must have non-negative value, if not then it can be converted into subtraction of two non-negative values.

Step4: The objective function should be of maximization form, if it is in the minimization form then convert it into maximization form as follows:

$$min\ Z = -max\ (-Z)$$

Example 5: Write the following LPP into its standard form:

$$max\ Z = 7x_1 + x_2 + 2x_3$$

Subject to:

$$x_1 + 4x_2 + x_3 \leq 5, \quad 3x_1 - x_2 + x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

Solution: Standard form of the LPP is:

$$Max\ Z = 7x_1 + x_2 + 2x_3 + 0S_1 + 0S_4$$

Subject to:

$$x_1 + 4x_2 + x_3 + S_1 = 5, \quad 3x_1 - x_2 + x_3 - S_4 = 8$$
$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

Where, S_1 and S_2 are non-negative slack and non-negative surplus variables respectively.

3. Quadratic Fractional Programming Problem

This section deals with quadratic fractional programming problem (QFPP) in which objective function is the ratio of quadratic numerator and linear denominator with linear inequality constraints and non-negative decision variables [6-7]. In 1965 Kanti Swarup[8] proposed an algorithm to solve LFPP without converting it, into the linear form. S.D. Sharma [9] gave a simplex algorithm for Quadratic Programming if the objective function is in the form of product of two linear functions (both are positive) and subject to the constraints are linear inequalities. Here we are introducing an algorithm for solving QFPP. Methodology of this algorithm is based upon mainly Kanti Swarup's fractional algorithm (KSFA) [8] and simplex method for quadratic programming.

3.1 Definition: The Mathematical form of QFPP is as follows:

$$\text{Max.} Z = \frac{(c_1^T X + \alpha)(c_2^T X + \beta)}{(d^T X + \gamma)}$$

Sub to:

$$AX \leq b$$
$$X \geq 0$$

Where,

(v) A is an $m \times n$ matrix,

(vi) b is an $m \times 1$ vector,

(vii) c_1, c_2, d and X are $n \times 1$ vectors,

(viii) c_1^T, c_2^T and d^T are transpose of c_1, c_2 and d respectively,

(ix) α and β are scalars,

(x) $d^T X + \beta > 0$ *and*

(xi) $S = \{X: AX \leq b, X \geq 0\}$ is non empty and bounded.

3.2 Notations: Following notations are used to solve QFPP:

$z^1 = c_{1B} X_B + \alpha$, $\quad z^2 = c_{2B} X_B + \beta$, $\quad z = d_B X_B + \gamma$,

$\Delta\xi_j^1 = c_{1B} x_j - c_{1j}$, $\quad \Delta\xi_j^2 = c_{2B} x_j - c_{2j}$, $\quad \Delta\xi_j = d_B x_j - d_j$,

$\mu_j = \left\{\frac{x_B}{x_j}, x_j > 0\right\}$ for non basic variables,

$\Delta_{1j} = z^2 \Delta\xi_j^1 + z^1 \Delta\xi_j^2 - \mu_j \Delta\xi_j^1 \Delta\xi_j^2$,

$\Delta_{2j} = \Delta\xi_j$,

$Z_1 = z^1 z^2$,

$Z_2 = z$,

$\Delta_j = Z_2 \Delta_{1j} - Z_1 \Delta_{2j}$,

$Z = \frac{Z_1}{Z_2}$.

3.3 Algorithm for Solving QFPP:

Step1: First of all, write the standard form of the given QFPP.

Step2: Construct the initial simplex table by using the notations (4.2).

Step3: Compute $\Delta_j = Z_2 \Delta_{1j} - Z_1 \Delta_{2j}$. Two cases will arise:

(i) If all $\Delta_j \geq 0$ then the optimal basic feasible solution is obtained.

(ii) If at least one $\Delta_j < 0$ then proceed on to the next step.

Step4: Choose

$$\Delta_r = \min.[\Delta_j, \Delta_j < 0]$$

then the corresponding variable of rth will enter the basis.

Step5: Find

$$\frac{x_{Bk}}{x_{kr}} = \min.\left\{\frac{x_{Bi}}{x_{ir}}, x_{ir} > 0, i = 1,2,\ldots,m\right\}$$

then the variable x_k will leave the basis .

Step6:
Convert the key element (i. e. x_{kr})into unity and all the other elements of the key column to zero by elementary ro operations.

Step7: Go to step3 and repeat the process until we get an optimal basic feasible solution.

3.4 Numerical Examples:

Example Solve

$$Max.Z = \frac{(2x_1 + 3x_2)(4x_1 + 6x_2 + 1)}{3x_1 + 2x_2 + 2}$$

Sub. to:

$$-2x_1 + x_2 \leq 1, \qquad 2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Solution: Standard form of the given LFPP is:

$$Max.Z = \frac{(2x_1 + 3x_2 + 0x_3 + 0x_4)(4x_1 + 6x_2 + 0x_3 + 0x_4 + 1)}{3x_1 + 2x_2 + 0x_3 + 0x_4 + 2}$$

Sub. To constraints

$$-2x_1 + x_2 + s_3 = 1, \quad 2x_1 + x_2 + s_4 = 8$$

$x_1, x_2, s_3, s_4 \geq 0.$

Where s_3 and s_4 are slack variables.Initial table:

Table-1

$\alpha = 0$				c_{1j}	2	3	0	0	
$\beta = 1$				c_{2j}	4	6	0	0	
$\gamma = 2$				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
s_3	0	0	0	1	−2	1	1	0	–
s_4	0	0	0	8	2	1	0	1	4
$z^1 = 0, \quad z^2 = 1$				$\Delta\xi_j^1$	−2	−3	0	0	
$z = 2$				$\Delta\xi_j^2$	−4	−6	0	0	
$\mu_1 = 4, \quad \mu_2 = 1$				$\Delta\xi_j$	−3	−2	0	0	
$Z_1 = 0$				Δ_{1j}	−26	−21	–	–	
$Z_2 = 2$				Δ_{2j}	−3	−2	0	0	
$Z = Z_1/Z_2 = 0$				Δ_j	−52	−42	–	–	

Drop s_4 and introduce x_1

Table 2

$\alpha = 0$				c_{1j}	2	3	0	0	
$\beta = 1$				c_{2j}	4	6	0	0	
$\gamma = 2$				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
s_3	0	0	0	9	0	2	1	1	9/2
x_1	2	4	3	4	1	1/2	0	1/2	8
$z^1 = 8, \quad z^2 = 17$				$\Delta\xi_j^1$	0	−2	0	1	
$z = 14$				$\Delta\xi_j^2$	0	−4	0	2	
$\mu_2 = 8, \quad \mu_4 = 9$				$\Delta\xi_j$	0	−1/2	0	3/2	
$Z_1 = 136$				Δ_{1j}	–	−130	–	15	
$Z_2 = 14$				Δ_{2j}	0	−1/2	0	3/2	
$Z = Z_1/Z_2 = \frac{136}{14}$				Δ_j	–	−2206	–	243	

Drop x_2 and introduce s_3

Table 3

$\alpha = 0$				c_{1j}	2	3	0	0	
$\beta = 1$				c_{2j}	4	6	0	0	
$\gamma = 2$				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
x_2	3	6	2	9/2	0	1	1/2	1/2	–
x_1	2	4	3	7/4	1	0	−1/2	1/4	–
$z^1 = 17, \quad z^2 = 35$				$\Delta\xi_j^1$	0	0	1/2	2	
$z = 65/4$				$\Delta\xi_j^2$	0	0	1	4	
$\mu_3 = 9, \quad \mu_4 = 7$				$\Delta\xi_j$	0	0	−1/2	7/4	
$Z_1 = 595$				Δ_{1j}	–	–	30	82	
$Z_2 = 65/4$				Δ_{2j}	0	0	−1/2	7/4	
$Z = Z_1/Z_2 = \frac{476}{13}$				Δ_j	–	–	78/5	1165/4	

Hence the basic feasible solution of the given problem is:

$$Max.Z = \frac{476}{13} = 36.61, \qquad x_1 = \frac{7}{4}, \qquad x_2 = \frac{9}{2}$$

5. Conclusion

We introduced a method to solve a linear fractional programming problem with interval coefficients in the objective function. In the proposed method, on using convex combination of the first and the last points of intervals instead of intervals and also using variable transformation, the initial problem is transformed into a nonlinear programming problem which finally is changed into a linear programming problem which has two more constraints and one more variable compare to the initial problem. The method is designed in such a way that each an every points of the intervals examined for achieving the best possible solution for the problem we gave an iterative procedure for solving linear fractional programming problems. Our procedure is based on the conjugate projection method for solving nonlinear programming problems with linear constraints. Starting with an initial interior point x^0, this procedure finds a sequence of feasible directions of movement such that each move increases the objective function until the optimal solution is found. Farther, we have defined Harmonic Average and Advanced Harmonic Average techniques and then compare Advanced Harmonic Average technique with other techniques namely Chandra Sen., Mean & Median, Arithmetic Mean & New Arithmetic Average, Geometric Mean & Advanced Geometric Average and Harmonic Average techniques. The comparisons of these techniques are based on the value of the objective functions. After solving the numerical example, we found that Max.Z which obtained by our technique (Advanced Harmonic average technique) is better than other techniques (Chandra Sen., mean & median, arithmetic mean & new arithmetic average, geometric mean & new geometric average and harmonic average techniques.

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