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Development of Mathematical Modelled Inventory System for the Deterioration of an Item with Two Warehouse and Shortage

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Abstract: *In this study, a two-warehouse inventory model with exponentially increasing trend in demand involving different deterioration rates under permissible delay in payment has been studied. Here, the scheduling period is assumed to be a variable. The objective of this study is to obtain the condition when to rent a warehouse and the retailer's optimal replenishment policy that minimizes the total relevant cost. An effective algorithm is designed to obtain the optimal solution of the proposed model. Numerical examples are provided to illustrate the application of the model. Based on the numerical examples, we have concluded that the single warehouse model is less expensive to operate than that of two warehouse model. Sensitivity analysis has been provided and managerial implications are discussed.*

Keywords: *Shortage, Inventory, Deteriorating, Warehouse, Imperfect items.*

1. Introduction

Deteriorating items are common in our daily life; however, academia has not reached a consensus on the definition of the deteriorating items.

According to the study of Wee HM in 1993 [1], deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons. The inventory problem of deteriorating items was first studied by Whitin [2], he studied fashion items deteriorating at the end of the storage period. Then Ghare and Schrader [3]concluded in their study that the consumption of the deteriorating items was closely relative to a negative exponential function of time. They proposed the deteriorating items inventory model as stated below:

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t)$$

In the function, θ stands for the deteriorating rate of the item, I(t) refers to the inventory level at time t and then f(t) is the demand rate at time t. This inventory model laid foundations for the follow-up study. Raafat [4] and Goyal and Giri [5] made comprehensive literature reviews on deteriorating inventory items in 1991 and 2001 respectively. From a different perspective, this paper reviews the recent trends in deteriorating inventory studies. Figure 1 presents the factors used to analyze and organize this review. From the perspective of scope, we make a distinction between the studies which focus on the deteriorating items inventory study in a single enterprise from those studies whose focus is on studying the deteriorating items inventory problems across a supply chain. The former is the focus of the early stage in the deteriorating items study and the latter now is attracting more and more attention from the researchers. From the perspective of the factors which should be taken into consideration in deteriorating items inventory study, we

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involve the important factors such as demand, deteriorating rate and other factors such as price discount, allow shortage or not, inflation, time-value of money and so on in our study. By way of integrating different factors in different scope (in a single enterprise or across a supply chain), different models can be established.

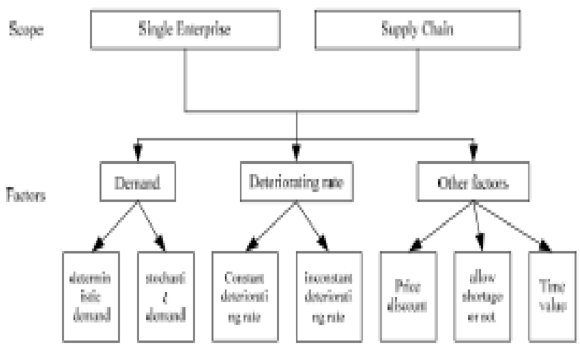

Figure 1. Deteriorating items inventory literature and its relation in the review

It is generally seen in real life conditions that there are so many reasons due to which the retailer intense to buy more than the capacity of the warehouse. In such cases the retailer needs an extra space to store the extra ordered quantity. This additional space is termed as rented warehouse. *Hartley (1976)* was the first to introduce a two-warehouse model. *Sarma (1983)* developed a deterministic inventory model for optimum release rule with infinite replenishment rate and two levels of storage [6]. *Pakkala and Achary (1992)* presented a deterministic inventory model for deteriorating items with two warehouses storage system and finite replenishment rate [7]. Yang (2004) introduced an inventory model for deteriorating items with two storage inventory system with shortages and time value of money. *Wee et al. (2005)* developed a two-storage inventory system with partial backlogging and Weibull distribution deterioration under inflation [8]. *Yang (2006)* considered a two-storage partial backlogging inventory models for deteriorating items under the rate of inflation [9]. *Singh et al. (2013)* discussed an inventory model with imperfect quality items with effect of earning and inflation under two finite storage capacity [10]. *Tayal et al. (2014)* introduced an inventory model for deteriorating products with space restriction. According to this model the quantity, extra from the storage capacity is returned to the supplier with a penalty cost [11]. *Rastogi et al. (2017)* presented a two warehouse inventory model with price sensitive demand and deterioration under shortages. To manage and control the inventory, for further procedure, which have deteriorating nature is very important. In last decades many researchers considered it and developed different inventory models [12]. *Ghare and Schrader (1963)* were the first to introduce the deterioration in inventory modelling [13]. *Covert and Philip (1973)* developed an inventory model with weibull rate of deterioration. *Chakrabarty et al. (1998)* extended this model with shortages and variable demand [14]. *Giri et al. (2003)* came forward with economics order quantity model with Weibull rate of deterioration, ramp type demand rate and allowable shortages [15]. *Skouri et al. (2009)* considered an inventory model for deteriorating items with ramp type demand rate and partial backlogging [16]. *Tayal et al. (2014)* presented an inventory model for multi items with variable rate of deterioration, expiration date and allowable shortages [17]. Further *Tayal et al. (2015)* developed an inventory model for deteriorating items with seasonal products and an option of an alternative market [18]. *Khurana et al. (2015)* introduced a supply chain production inventory model for deteriorating product with stock dependent demand under inflationary environment and partial backlogging [19].

Tayal et al. (2015) came forward with a production model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate [20]. *Singh et al. (2016)* discussed an inventory policy for deteriorating product with seasonal and stock level dependent demand with partial backlogging of shortages [21]. *Tayal et al. (2016)* presented an integrated production inventory model for perishable products with trade credit period and preservation technology [22]. Singh et al. (2016) came forward with an economic order quantity model for deteriorating products with preservation having stock dependent demand and trade credit period [23]. *Rastogi et al. (2017)* developed an EOQ policy with time dependent holding cost and partial backlogging under trade credit limit and cash discount policy [24]. *Pandey et al. (2017)* considered an EOQ model with quantity incentive strategy for deteriorating items and partial backlogging [25]. *Khurana et al. (2018)* developed an economic production quantity model for deteriorating product with variable demand rate and allowable shortages. Inflation plays a very important role in the development of an inventory model. The models developed without considering the inflation in price misleads the results [27]. *Buzacott (1975)* was the first to develop an inventory model as economic order quantities with inflation [28]. *Teng et al. (1999)* considered a deterministic inventory model with shortages and deterioration for fluctuating demand under the inflationary environment to find out the economic lot size [29]. *Wee et al. (2005)* introduced a two storage inventory model with partial backlogging during stock out and weibull rate of deterioration under inflation. *Sarkar and Moon (2011)* developed a production quantity model considering inflation in an imperfect production system [30]. *Tayal et al. (2014)* presented an inventory model with two echelon supply chain for deteriorating items considering preservation technology investment [30].

2. Inventory Modeling

We need a lot of things to survive in our daily lives. All of us are aware of our current demands, but frequently, we are unsure of our future needs. The items we use every day are either bought from retail stores or occasionally from wholesale warehouses. Inventory is the collection of items created to satisfy immediate and future needs. In other terms, "Inventory is defined as a stock of things held on hand by a corporation to use in meeting consumers' present-day and near-term demand. It is seen as an economy of a firm's idle resource. Inventory is crucial to a business from both a financial and an operational standpoint. First off, any business must make a significant financial investment in its inventory. However, from an operational standpoint, inventories increase operational flexibility. Production is streamlined in manufacturing organisations by maintaining adequate supplies. By maintaining adequate inventory, wholesalers and retailers may provide good customer service and improve their reputation. Thus, achieving a balance between minimal inventory and good return on investment is the primary goal of inventory management.

The purpose of all inventory models is to minimize inventory costs. As a result of the inventory model, a designer of air-condition machine decided to redesign its old model machine to enhance its working efficiency and reduce inventory costs in meeting a global market for it air-condition machines. The purpose of modeling inventory situations is to derive an operating doctrine which follows four steps mentioned below:

• Inventory situation should be carefully examined and characteristics should be listed with assumptions concerning the situation.
• The total annual relevant cost equation should be developed in narrative.
• The total annual cost equation should be transformed from narrative into the shorthand logic of mathematics.
• The cost equation should be optimized and the optimum solution is obtained under given assumption and restriction for how much to order quantity and when to reorder (cycle length).

After the completion of the above process, the objective is to find the values of decision variables that optimize the cost function. In searching optimal solution for the objective function so modeled a feasible solution (The set of values of decision variables that satisfies all the constraints) is determined using mathematical tools/computer software and thereafter the optimal value (the best possible objective function value) for decision variables is searched within feasible solutions.

2.1 Inventory Control Techniques

A company's large number of stored inventories may be difficult to manage, and it is also difficult to analyse client demand to determine order frequencies and anticipated demand rates with equal care. The firm's possessions may not all be equally valuable for sale. It is vital to divide the various stock kinds into groups according to the level of management effort or control they require in order to provide priority to sales. As a result, inventories are divided into the following groups:

• ABC (Always, Better, Control)
In this technique the inventory is classified on the basis of consumption.
• HML (High, Medium, Low)
In this technique the inventory is classified on the basis of unit price.
• XYZ
In this technique the inventory is classified on the basis value of items in storage.
• VED (Vital, Essential, Desirable)
In this technique the inventory is classified on the basis of criticality of components of an item included in inventory.
• FSN (Fast, Slow, Non-moving)
In this technique the inventory is classified on the basis of consumption pattern of the component.
• SDE (Scare, Difficult, Easy to obtain)
In this technique the inventory is classified on the basis of problem faced in procurement.
• GOLF (Government, Ordinary, Local, Foreign)
In this technique the inventory is classified on the basis of source of material.
• SOS (Seasonal, Off-Seasonal)

In this technique the inventory is classified on the basis of nature of supply of the products/materials.

3. Mathematical Modelling

3.1 Assumption and Notation

The mathematical model for the two-warehouse inventory problem is developed on the basis of the following assumptions;

Assumptions

- Demand rate is constant and known.
- The lead time is negligible and initial inventory level is zero.
- The replenishment rate is infinite.
- Shortages are allowed and partially backordered.
- Deterioration rate in both the warehouses is different.
- Deterioration rate is time dependent and follows a

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two parameter weibull distribution where $a > 0$ denote scale parameter and $p > 1$ denote the shape parameter.

- The salvage value $y(0 < y < 1)$ is associated to deteriorated units during the cycle time.
- The holding cost is a linear function of time and is higher in RW than OW.
- The holding cost is a linear function of time and is higher in RW than OW.
- Deterioration occurs immediately after receiving the items into inventory.

Notations

The following notations are used throughout this chapter:

d Demand rate units/unit time (constant)
W Capacity of OW
a Scale parameter of the deterioration rate in OW and $0 < a < 1$
P Shape parameter of the deterioration rate in OW and > 1
p Scale parameter of the deterioration rate in RW, a > p, 0<p<1
n Shape parameter of the deterioration rate
F Fraction of the demand backordered during the stock out period
C_0 Ordering cost per order
C_d Deterioration cost per unit of deteriorated item in both warehouses
$H_o=bt_1$:Holding cost per unit per unit time in OW during T_1 time period; $b>0$
$H_o=bt_2$:Holding cost per unit per unit time in OW during T_2 time period; $b>0$
$H_R=at_x$:Holding cost per unit per unit of time in RW during T_1 time period where
$H_R > H_o$
C_s Shortage cost per unit per unit of time
L_c Shortage cost for lost sales per unit per unit of time
Qo The order quantity in OW
Q_R The order quantity in RW
QM Maximum ordered quantity after a complete time period T
I_k Maximum inventory level in RW
T_1 Time with positive inventory in RW
T_1+T_2 Time with positive inventory in OW
T_3 Time when shortage occurs in OW
T Length of the cycle, $T = T_1 + T_2 + T_3$
iP(t,) Inventory level in OW at time t, $0 < t_t$
I% (tj) Inventory level in RW at time t_j, $0 < t_1$
T The present value of the total relevant inventory cost per unit time
The rate of deterioration is given as follows: t

Time to deterioration, $t > 0$
Instantaneous rate of deterioration in OW Z (t) = $apt^{)-1}$
where $0 < a < 1$
Instantaneous rate of deterioration in RW R (t) =
where $0 < p < 1$

Two warehouse system Mathematical Modelling

Figure-3.1 represents the inventory system of OW. It can be divided into three part depicted by T_1, T_2 and T_3. For each replenishment, a portion of the replenished quantity is used to backlog shortage, while the rest enters into the system. W units of items are stored in OW and the rest are kept into RW. The inventory level in RW inventory system has been depicted graphically in Figure-3.2.

The inventory in RW is supplied first to reduce the inventory cost due to more holding cost as compared to OW. Stock in the RW during time interval T_1 depletes due to demand and deterioration until it reaches zero. During the time interval, the inventory in OW decreases due to deterioration only. The stock in OW depletes due to the combined effect of demand and deterioration during time T2. During the time interval T3, both warehouses are empty, and part of the shortage is backordered in the next replenishment.

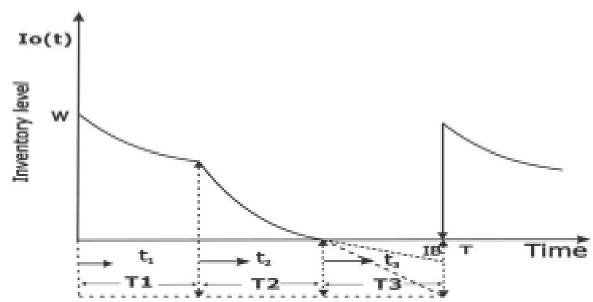

Figure-2 Graphical representation for OW Inventory System

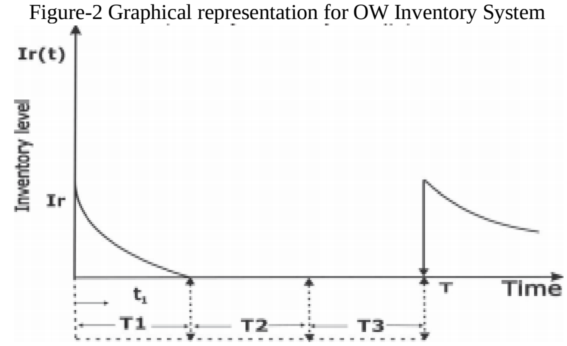

Figure-3 Graphical representation for RW Inventory System (Inventory Level vs. Cycle Length)

$< T_i, i = 1,2,3$
$< T_1$

In the first stage of inventory cycle, demand of customers is fulfilled from the items stored in RW so the rate of change of inventory during positive stock in RW in time period T_1 is given by the following differential equation

$$\frac{dI^R(t_1)}{dt_1} = -\mu\eta t_1^{\eta-1} I^R(t_1) - d; \qquad 0 \le t_1 \le T_1$$

Solution of above equation with B.C. $I^R(0) = I^r$ is

$$I^r(t_1) = \left(I^r - d\int_0^{t_1} e^{\mu u^\eta} du\right) e^{-\mu t_1^\eta}; \quad 0 \le t_1 \le T_1$$

$$Where\ I^r = d\int_0^{T_1} e^{\mu u^\eta} du = d\sum_{m=0}^{\infty} \frac{\mu^m T_1^{m\eta+1}}{m!(m\eta+1)}$$

$$\approx d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right)$$

The rate of change of inventory during positive stock in OW and time period $T_1+T_2+T_3$ can be represented by the following differential equation

$$\frac{dI_1^O(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} I_1^O(t_1); \qquad 0 \le t_1 \le T_1 \qquad (3.4)$$

$$\frac{dI_2^O(t_2)}{dt_2} = -\alpha\beta t_1^{\beta-1} I_2^O(t_2) - d; \qquad 0 \le t_2 \le T_2 \qquad (3.5)$$

Shortges starts during the stock out time period T_3 in OW and can be represented by the differential equation

$$\frac{dI_3^O(t_3)}{dt_3} = -Fd; \qquad 0 \le t_3 \le T_3 \qquad (3.6)$$

Solution of above differential equation with boundary conditions $I_1^0(0) = W$,

$I_1^O(T_1) = We^{-\alpha T_1^\beta} = I_2^O(0)$ and $I_3^O(0) = 0$ can be given as

$$I_1^O(t_1) = We^{-\alpha t_1^\beta}; \qquad 0 \le t_1 \le T_1 \qquad (3.7)$$

$$I_2^O(t_2) = \left(We^{-\alpha T_1^\beta} - d\int_0^{t_2} e^{\alpha u^\beta} du\right) e^{-\alpha t_2^\beta}; \qquad 0 \le t_2 \le T_2 \qquad (3.8)$$

$$I_3^O(t_3) = -Fdt_3; \qquad 0 \le t_3 \le T_3 \qquad (3.9)$$

The amount of inventory deteriorated during time period T_1 in RW is denoted by D^R and is given as

$$D^R = \int_0^{T_1} R(t) I^R(t) dt_1$$

$$= \int_0^{T_1} \mu\eta\, t^{\eta-1} \left(I^r - d\int_0^{t_1} e^{\mu u^\eta} du\right) e^{-\mu t_1^\eta} dt_1$$

$$= \mu d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right) T_1^\eta \approx \mu\, d\, T_1^{\eta+1} \qquad (3.10)$$

Cost of deteriorated items in RW is denoted and given as

$CD^R = C_d \mu d T_1^{n+1}$ ……………………………(3.11)

The amount of inventory deteriorated during time period $T_1 + T_2$ is OW is denoted by D^0 and is given as

$$D^0 = \int_0^{T_1} Z(t)\, W dt_1 + \left(We^{-\alpha T_1^\beta} - \int_0^{T_2} d\ dt_2\right)$$

$$= \int_0^{T_1} \alpha\beta t^{\beta-1}\ W dt_1 + \left(We^{-\alpha T_1^\beta} - \int_0^{T_2} d\ dt_2\right)$$

$$= W\left(\alpha T_1^\beta + e^{-\alpha T_1^\beta}\right) - dT_2 \qquad (3.12)$$

Cost of deteriorated items in OW is denoted by CD^0 and is given as

$$CD^0 = C_d\left\{W\left(\alpha T_1^\beta + e^{-\alpha T_1^\beta}\right) - dT_2\right\} \qquad (3.13)$$

The maximum ordered quantity is denoted by M_Q and is given as

$$M_Q = d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right) + W + Fd\ \frac{T_3^2}{2} \qquad (3.14)$$

The inventory holding cost in OW durign time period $T_1 + T_2$ is denoted by IH^0 and is given as

$$IH^0 = \int_0^{T_1} H_0\ I_1^O(t_1)\, dt_1 + \int_0^{T_2} H_0\ I_2^O(t_2)\, dt_2$$

$$= \int_0^{T_1} bt_1\ We^{-\alpha t_1^\beta}; dt_1 + \int_0^{T_2} bt_2 \left(We^{-\alpha T_1^\beta} - d\int_0^{t_2} e^{\alpha u^\beta} du\right) e^{-\alpha t_2^\beta} dt_2$$

$$= \left[bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + bW\left\{\frac{T_2^2}{2}\left(1 - \alpha T_1^\beta\right) - \frac{\alpha T_2^{\beta+2}}{\beta+2}\right\} - bd\left\{\frac{T_2^3}{3} - \frac{\alpha\beta T_2^{\beta+3}}{(\beta+1)(\beta+3)}\right\}\right] \qquad (3.15)$$

Shortages occurs during time period T_3 due to non-availability of stock in OW, which is denoted by S_H and is given as

$$S_H = \int_0^{T_3} \{-\ I_3^O(t_3)\}\, dt_3$$

$$= \int_0^{T_3} \{-\ (-Fdt_3)\ \}\, dt_3$$

$$= Fd\ \frac{T_3^2}{2}$$

…………(3.16)

Shortages cost of inventory short is denoted and given as

$$CS_H = C_S\ Fd\ \frac{T_3^2}{2}$$

……………………………..(3.17)

Lost sales occurs during shortages period in OW due to partial backlogging and the amount of sale lost is denoted by L_S and is given as

$$L_S = \int_0^{T_3} (1-F) d\ \ dt_3$$

$$= (1-F) d\ T_3$$

…………………(3.18)

Cost of lost sales is denoted by CL_S and is given as

$CL_s = L_C(1–F)\ d\ T_3$……………………………….(3.19)

The inventory holding cost in RW during time period T_1 is denoted by IH^R and is given as

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$$IH^R = \int_0^{T_1} at_1\, I^R(t_1)\, dt_1$$

$$= \int_0^{T_1} at_1\, (I^r - d\int_0^{t_1} e^{\mu u^{\eta}} du) e^{-\mu t_1^{\eta}}\, dt_1$$

$$= ad\left\{\frac{T_1^3}{6} + \frac{\mu\eta T_1^{\eta+3}}{2(\eta+2)(\eta+3)}\right\} \tag{3.20}$$

where $I^r = d\int_0^{T_1} e^{\mu u^{\eta}} du = d\sum_{m=0}^{\infty} \frac{\mu^m T_1^{m\eta+1}}{m!(m\eta+1)}$

The salvages cost of deteriorated units per unit time is denoted by SV and is given as

$$SV = \gamma\left[\mu\, d\, T_1^{\eta+1} + W(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}) - dT_2\right] \tag{3.21}$$

The present value of the total inventory cost during the cycle denoted by T_C^1 is the sum of ordering (OC) per cycle, the inventory holding cost (IH^R) per cycle in RW, the inventory holding (IH^o) per cycle in OW, Deterioration cost per cycle in RW, Deterioration cost per cycle OW, the shortages cost (CH_H) in OW, the lost sales cost (CL_S) and minus the salvages value of deteriorated units i.e.

$$T_C^1(T_1, T_2, T_3) = \frac{1}{T}[OC + IH^R + IH^O + CD^R + CD^O + CS_H + CL_S - SV]$$

$$= \frac{1}{T}\Bigg[O_c + ad\left\{\frac{T_1^3}{6} + \frac{\mu\eta T_1^{\eta+3}}{2(\eta+2)(\eta+3)}\right\} + \left[bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + bW\left\{\frac{T_2^2}{2}\left(1 - \alpha T_1^{\beta}\right) - \frac{\alpha T_2^{\beta+2}}{\beta+2}\right\} -\right.$$

$$\left. bd\left\{\frac{T_2^3}{3} - \frac{\alpha\beta T_2^{\beta+3}}{(\beta+1)(\beta+3)}\right\}\right] + C_d\, \mu\, d\, T_1^{\eta+1} + C_d\left\{W\left(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}\right) - dT_2\right\} + C_S\, Fd\, \frac{T_3^2}{2} +$$

$$L_C(1-F)d\, T_3 - \gamma\left[\mu\, d\, T_1^{\eta+1} + W(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}) - dT_2\right]\Bigg] \tag{3.22}$$

The optimal problem can be formulated as

Minimize : $T_C^1(T_1, T_2, T_3)$

Subject to : $T_1 \geq 0, T_2 \geq 0, T_3 \geq 0$

To find the optimal solution of the equation the following conditions must be satisfied

$$\frac{\partial T_C^1(T_1, T_2, T_3)}{\partial T_1} = 0; \frac{\partial T_C^1(T_1, T_2, T_3)}{\partial T_2} = 0; \frac{\partial T_C^1(T_1, T_2, T_3)}{\partial T_3} = 0 \tag{3.23}$$

And the Hessian matrix given below must be a positive semi-definite in the domain for global minima.

$$\begin{pmatrix} \frac{\partial^2 T_C^1}{\partial T_1 \partial T_1}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_1 \partial T_2}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_1 \partial T_3}(\check{T}) \\ \frac{\partial^2 T_C^1}{\partial T_2 \partial T_1}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_2 \partial T_2}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_2 \partial T_3}(\check{T}) \\ \frac{\partial^2 T_C^1}{\partial T_3 \partial T_1}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_3 \partial T_2}(\check{T}) & \frac{\partial^2 T_C^1}{\partial T_3 \partial T_3}(\check{T}) \end{pmatrix} \text{ where } \check{T} = (\check{T}_1, \check{T}_2, \check{T}_3)$$

Solving equation (3.23) for T_1, T_2 and T_3 respectively the values of T_1, T_2, T_3 and T^* are obtained and these values are used to find total minimum inventory cost from equation (3.22)

Single Ware-house System

Figure-3.3 shows the graphical representation of one ware-house inventory system. Considering a one warehouse inventory system we derived the inventory level during time periods T_1 and T_2 which is represented by the following differential equation

$$\frac{dI_1^O(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} W - d; \qquad 0 \leq t_1 \leq T_1 \tag{3.24}$$

Solution of above differential equation with boundary conditions, $I_1^0(0) = W$

$$I_1^O(t_1) = W(1 - \alpha t_1^{\beta}) - d\, t_1 \qquad 0 \leq t_1 \leq T_1 \tag{3.25}$$

Shortages occur during the time perod [0, T_2]. The present worth shortages cost is

$$S_C = C_S\{\int_0^{T_2} (Fd\, t_2)\, dt_2$$

$$= \frac{C_S\, Fd}{2} T_2^2 \tag{3.26}$$

Loss of sales occur during T_2 time period. The OW present worth lost sales cost is given as

$$CL_S = L_C\{\int_0^{T_2} (1 - F)d\, dt_2\}$$

$$= L_C\, (1 - F)d\, T_2 \tag{3.27}$$

Cost of Deteriorated unites in time interval [0 T_1] is given as

$$CD^R = C_d\, \alpha W\, d\, T_1^{\beta} \tag{3.28}$$

The maximum order quantity per order is

$$M_Q = W + \frac{Fd}{2} T_2^2 \tag{3.29}$$

Salvages value of deteriorated units per unittime is

$SV = \gamma W \alpha T_1^{\beta}$ …………………………………. (3.30)

Inventory holding cost during time period T_1 is

$$IH^O = \int_0^{T_1} H_O\, I_1^O(t_1)\, dt_1$$

$$= \int_0^{T_1} bt_1\, (W(1 - \alpha t_1^{\beta}) - d\, t_1)\, dt_1$$

$$= bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\}$$

Nothing that $T = T_1 + T_2$ the total present value of the total relevant cost per unit time during the cycle is the sum of the ordering cost ,holding cost shortages cost ,lost sales cost minus salvages value of deteriorated units i.e.

$$T_C^1(T_1, T_2) = \frac{1}{T}\left[\begin{array}{c} O_c + bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + C_d\, \alpha W\, d\, T_1^{\beta} + C_S\, Fd\, \frac{T_2^2}{2} \\ + L_C(1-F)d\, T_2 - \gamma W \alpha\, T_1^{\beta} \end{array}\right] \tag{3.32}$$

The optimal problem can be formulated as

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Minimize: $T_C^I\,(T_1, T_2)$

Subject to: $T_1 \geq 0,\ T_2 \geq 0$;

To find the optimal solution the following condition must be satisfied

$$\frac{\partial T_C^I(T_1,T_2)}{\partial T_1}=0; \qquad \frac{\partial T_C^I(T_1,T_2)}{\partial T_2}=0; \tag{3.33}$$

Provided

$$\left(\frac{\partial^2 T_C^I(T_1,T_2)}{\partial T_1^2}\right)\left(\frac{\partial^2 T_C^I(T_1,T_2))}{\partial T_2^2}\right)-\left(\frac{\partial^2 T_C^I(T_1,T_2)}{\partial T_1 \partial T_2}\right)^2 > 0 \text{ and}$$

$$\left(\frac{\partial^2 T_C^I(T_1,T_2)}{\partial T_1^2}\right) > 0 \,;\left(\frac{\partial^2 T_C^I(T_1,T_2))}{\partial T_2^2}\right) > 0 \; at\, (\breve{T}_1, \breve{T}_2)$$

Solvng equations (3.33) for T_1, T_2 respectively the values of $\breve{T}_1$, $\breve{T}_2$ and T* are obtained and these values are used to calculated the total minimum inventory cost from equation (3.32)

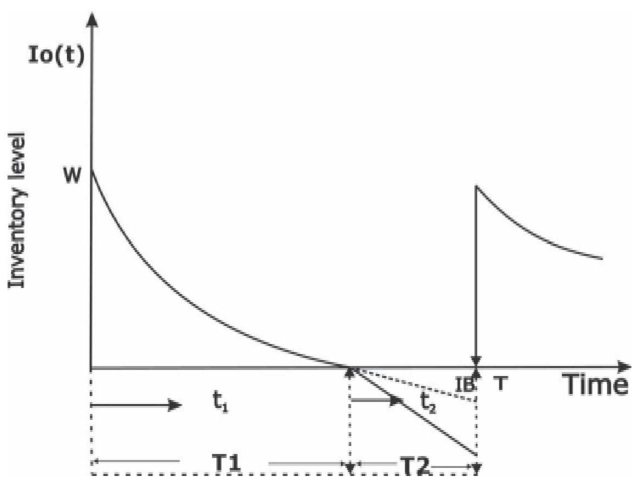


Figure-3 Graphical representation for a single warehouse Inventory System
(Inventory Level Vs. Cycle Length)

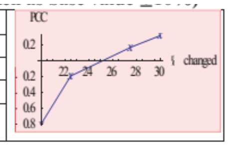
Sensitivity Analysis

In order to study the effect of change in the value of parameters after the optimal solution, sensitivity analysis is performed for the numerical example. The optimal values of T_1*, T_2*, T_3* and T£* are derived when one of the parameters in the subset S increases by 10 % and all other parameters remain unchanged. The result of the sensitivity analysis are shown in Table-3.2 with corresponding graphical representation. The change in the total relevant inventory cost given as percentage Change in Cost (PCC) is given by

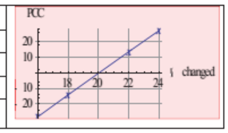
$$PCC=\frac{T_C^{IC}-T_C^{I*}}{T_C^{I*}}\times 100$$

Table-1: Representing Sensitivity Analysis with respect to parameters to study percentage change in cycle length and cost function (Data of numerical example is taken as base value ±10%)

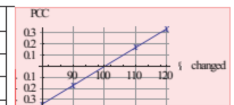
a	$\breve{T}_{1*}$	$\breve{T}_{2*}$	$\breve{T}_{3*}$	T_C^{IC}	PCC (%)
30	0.3732	3.4991	0.0866	1492.72	0.32
27.5	0.3859	3.5002	0.0863	1490.43	0.17
22.5	0.4169	3.5029	0.0856	1485.08	-0.19
20	0.4362	3.6668	0.0853	1475.85	-0.80



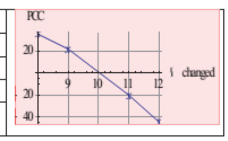
b	$\breve{T}_{1*}$	$\breve{T}_{2*}$	$\breve{T}_{3*}$	T_C^{IC}	PCC (%)
24	0.4515	3.5220	0.1370	1896.03	27.43
22	0.4268	3.5123	0.1116	1692.90	13.78
18	0.3714	3.4894	0.0607	1280.54	-13.94
16	0.3393	3.4756	0.0338	1070.75	-28.04



C_o	$\breve{T}_{1*}$	$\breve{T}_{2*}$	$\breve{T}_{3*}$	T_C^{IC}	PCC (%)
120	0.4008	3.5002	0.0863	1492.90	0.34
110	0.4013	3.4989	0.0866	1490.39	0.17
90	0.3998	3.5027	0.0857	1485.38	-0.17
80	0.3993	3.5040	0.0853	1482.87	-0.34



C_d	$\breve{T}_{1*}$	$\breve{T}_{2*}$	$\breve{T}_{3*}$	T_C^{IC}	PCC (%)
12	0.2515	3.4328	0.0024	818.951	-44.96
11	0.3307	3.4653	0.0448	1185.25	-20.34
9	0.4636	3.5402	0.1262	1809.35	21.61
8	0.5224	3.5807	0.1655	2012.73	35.28



The following conclusions are made from Table-1:

(1) The value of PCC is the highly sensitive to the parameters W (capacity of own ware-house), Y (Salvages value incurred on deteriorated items), b (Holding cost of inventory in OW and is directly proportional to these values.

(2) The Value of PCC is the highly sensitive to the C_d (Cost of deterioration) and sensitive to d (Demand of inventory) a (Scale parameter of the deterioration rate in OW), and is indirectly proportional to these values.

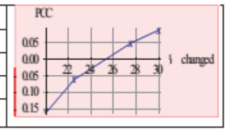
(3) The value of PCC is slightly sensitive to the values of P (The shape parameter of deterioration rate in RW, p (scale parameter of deterioration rate in OW), C_s (Cost of shortages), L_c (Cost of lost sale), Co (Ordering cost), a (Holding cost in RW) and is directly proportional to these values.

(4) The value of PCC is slightly sensitive to the values of F (Amount of shortages backlogged) and is indirectly proportional to it.

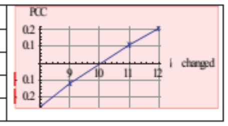
(5) The value of PCC is not sensitive to the value of " (Shape parameter of deterioration rate in RW).

The graphical representation of the changes in the value of PCC with corresponding change in the one parameter keeping others unchanged is shown in the above table.

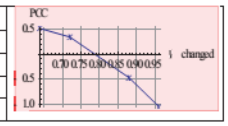
C_s	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC(%)
30	0.4006	3.5009	0.0718	1489.18	0.09
27.5	0.4005	3.5012	0.0782	1488.56	0.05
22.5	0.4002	3.5019	0.0954	1487.06	-0.06
20	0.3999	3.5024	0.1072	1486.04	-0.12



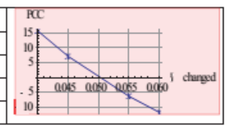
L_s	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
12	0.4009	3.4999	0.0664	1490.95	0.21
11	0.4006	3.5067	0.0762	1489.51	0.11
9	0.3995	3.5024	0.0958	1486.09	-0.12
8	0.3996	3.5034	0.1055	1484.05	-0.26



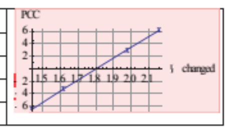
F	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
0.96	0.3973	3.5092	0.1367	1472.48	-1.04
0.88	0.3989	3.5050	0.1137	1480.88	-0.47
0.72	0.4013	3.4989	0.0518	1492.99	0.34
0.64	0.4018	3.4977	0.0087	1495.50	0.51



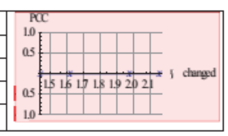
α	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC(%)
0.060	0.3653	3.1908	0.0644	1315.15	-11.61
0.055	0.3818	3.3357	0.0744	1395.59	-6.20
0.045	0.4215	3.6935	0.0940	1595.21	7.21
0.040	0.4459	3.9198	0.1153	1722.08	15.74



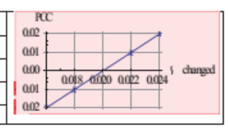
β	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC(%)
2.16	0.4159	3.6657	0.0975	1580.07	6.20
1.98	0.4083	3.5846	0.0918	1534.49	3.13
1.62	0.3921	3.4163	0.0800	1440.17	-3.19
1.44	0.3835	3.3288	0.0739	1391.29	-6.49



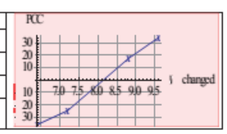
η	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
2.16	0.4003	3.5015	0.0859	1487.88	0.00
1.98	0.4003	3.5015	0.0859	1487.88	0.00
1.62	0.4003	3.5015	0.0859	1487.88	0.00
1.44	0.4003	3.5015	0.0859	1487.88	0.00



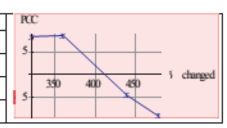
μ	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
0.024	0.3997	3.5013	0.0806	1488.20	0.02
0.022	0.4000	3.5014	0.0860	1488.04	0.01
0.018	0.4006	3.5016	0.0859	1487.72	-0.01
0.016	0.4009	3.5017	0.0859	1487.56	-0.02



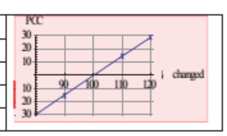
Y	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
9.6	0.4994	3.5643	0.1499	1998.77	34.33
8.8	0.4515	3.5323	0.0118	1745.65	17.32
7.2	0.3452	3.4725	0.0531	1124.59	-24.41
6.4	0.2846	3.4452	0.0195	955.95	-35.75



D	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
480	0.3475	3.4884	0.0410	1353.68	-9.02
440	0.3726	3.4945	0.0616	1421.80	-4.44
360	0.4452	3.5176	0.1525	1615.73	8.59
320	0.4669	3.5195	0.1519	1611.91	9.34



W	T_{1*}	T_{2*}	T_{3*}	T_C^{IC}	PCC (%)
120	0.4556	3.5297	0.1396	1917.06	28.85
110	0.4289	3.5164	0.1129	1703.62	14.50
90	0.3690	3.4854	0.0587	1269.66	-14.67
80	0.3342	3.4649	0.0311	1048.43	-29.54



4. Conclusion

An inventory model helps to identify the best replacement cycle for two warehouse inventory problems with varied rates of wear and partial backlogs. The model presupposes that the distributors' warehouse has a finite capacity. By minimizing the total relevant cost of the inventory system, the optimal replenishment policy is determined using the optimization technique. An example using numbers is provided to demonstrate the model's viability. The overall relevant cost per unit time of the inventory system is higher when just one warehouse is present than when there are two warehouses. This approach works well for items that degrade quickly according to the Weibull distribution since inventory costs are indirectly proportional to demand. It is observed that the model is highly sensitive to the inflation rate and advertising cost and indirectly proportional to these parameters. The model is slightly sensitive to holding cost in RW, ordering cost, and deterioration rate in both warehouses and moderately sensitive to other parameters. The ordering cycle length is moderately sensitive to all parameters and highly sensitive to the lost sale cost. The model is highly sensitive to opportunity lost cost sale and the demand parameter. Backlogs of shortages are in part cleared. The model accounts for time-dependent advertising costs. Since inventory costs decrease when inflation and advertising spending rise, the model is realistically beneficial.

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