



A Review on Thermal Temperature uncertainty in an Oldroyd-B nanofluid saturated Absorbent layer

Kirti Tiwari¹ and Dr. Arihant Jain²

Department of Mathematics, Research Scholar, Dr. A.P.J. Abdul Kalam University, Indore¹

Department of Mathematics, Research Supervisor, Dr. A.P.J. Abdul Kalam University, Indore²

Abstract: *By taking into account the effects of Brownian diffusion and thermophoresis, the start of convective instability in a layer of porous material saturated by the Oldroyd-B viscoelastic nanofluid heated from below is explored. On the boundary, it is assumed that the flux of the nanoparticle volume fraction is zero. The Galerkin method is used to numerically solve the resulting eigenvalue problem. When the strain retardation parameter does not surpass a threshold value that in turn depends on other physical parameters, the start of convective instability is only oscillatory if the strain retardation parameter is less than the stress relaxation parameter. When the strain retardation parameter is increased, the oscillatory beginning is delayed; however, when the stress relaxation parameter is increased, the opposite trend is seen. The onset of stationary and oscillatory convection is accelerated by increasing the modified diffusivity ratio, concentration Darcy-Rayleigh number, modified particle density increment, and Lewis number. Additionally, the ranges of the strain retardation parameter within which oscillatory convection is preferred are reduced.*

Keywords: *Galerkin method, Modulation, Dual diffusion, Convection, Darcy-Rayleigh number.*

1. Introduction

Recently, there has been considerably significant interest in nanofluids. This interest in nanofluids has been generated due to the wide variety of applications, ranging from laser-assisted drug delivery to electronic chip cooling. Nanoparticles (with sizes typically in the range between 1 and 100 nm) suspended in a base fluid, which can be water or an organic solvent (Ganguly et al. [1], Merabia et al. [2]) are referred to as nanofluids. Nanofluids in porous media constitute an emerging topic; the review of recent literature points out to at least two possible applications. Porous foam and micro channel heat sinks (used for electronic cooling) are usually modeled and optimized utilizing the porous medium approach (Kim et al. [3], Kim and Kuznetsov [4]). The double diffusive convection can be termed as a problem of convection induced by temperature and concentration gradients or by concentration gradients of two species. Whenever two diffusing properties are present in a system, then instabilities can occur only if one of the components are destabilizing. Although the problem of onset of convection has been extensively investigated for Newtonian fluids, relatively little attention has been

devoted to the thermal convection of viscoelastic fluids (see, e.g., Li and Khayat [5] and references there in). From a rheological point of view, the study of onset of convection in viscoelastic fluid may be important because the observation of the onset of convection provides potentially useful techniques to investigate the suitability of a constitutive model adopted for a certain viscoelastic fluids. Recently the linear stability of viscoelastic fluid saturated porous layer using a thermal non-equilibrium model by considering the Oldroyd-B type fluid have been studied by Malashetty et al. [6].

The interest in the study of convective instability of viscoelastic fluids saturating a porous layer is of two fold. On the one hand, the influence of the viscoelastic properties however small can lead to important changes in the convective pattern. On the other hand, the study can be a useful tool to determine some rheological properties of a given fluid. In the present research paper, we intend to study the onset of double diffusive convection in a viscoelastic nano fluid saturated porous layer heated from below with emphasis on how the condition for the onset of convection is modified by the elastic effects. The Oldroyd



model, which allows fitting the data of many polymeric solutions, is employed to include the viscoelastic properties, as this model is more general in nature compared to Maxwell and Jeffrey's models.

Nomenclature			
a	wave number	ϵ	porosity of porous media
D_B	Brownian diffusion coefficient	η	thermal expansion coefficient of viscosity
D_T	thermophoretic diffusion coefficient	κ	thermal diffusivity of the fluid
d	depth of the porous layer	λ_1	constant relaxation time
k	thermal conductivity of the nanofluid	λ_2	constant retardation time
K	permeability of the porous medium	Λ_1	stress relaxation parameter
Le	Lewis number	Λ_2	strain retardation parameter
l, m	wave numbers in the x - and y -directions	μ	viscosity of the fluid
M	heat capacity ratio	ω	growth rate
N_A	modified diffusivity ratio	ϕ	nanoparticle volume fraction
N_B	modified particle density increment	ϕ_0	Reference value of nanoparticle volume fraction
p	pressure	Φ	amplitude of perturbed nanoparticle volume fraction
$\vec{q} = (u, v, w)$	nanofluid velocity	ρ	nanofluid density
R_m	basic density Darcy-Rayleigh number	Θ	amplitude of perturbed temperature
R_T	thermal Darcy-Rayleigh number		
R_c	nanoparticle concentration Darcy-Rayleigh number		
(x, y, z)	Cartesian coordinates	Superscripts	
t	time	*	dimensionless variable
T	nanofluid temperature	\prime	perturbed variable
T_0	temperature at the lower boundary	Subscripts	
T_1	temperature at the upper boundary	b	basic state
W	amplitude of perturbed vertical component of velocity	f	fluid
		p	particle
Greek symbols			
β	the coefficient of thermal expansion		

2. Double diffusive convection

Double diffusive convection is a fluid dynamics phenomenon that describes a form of convection driven by two different density gradients, which have different rates of diffusion.[7] Convection in fluids is driven by density variations within them under the influence of gravity. These density variations may be caused by gradients in the composition of the fluid, or by differences in temperature (through thermal expansion). Thermal and compositional gradients can often diffuse with time, reducing their ability to drive the convection, and requiring that gradients in other regions of the flow exist in order for convection to continue. A common example of double diffusive convection is in oceanography, where heat and salt concentrations exist with different gradients and diffuse at differing rates. An effect that affects both of these variables is the input of cold freshwater from an iceberg. A good discussion of many of these processes is in Stewart Turner's monograph "Buoyancy effects in fluids".[8]

Double diffusive convection is important in understanding the evolution of a number of systems that have multiple causes for density variations. These include convection in the Earth's oceans (as mentioned above), in magma chambers,[9] and in the sun (where heat and helium diffuse at differing rates). Sediment can also be thought as having a slow Brownian diffusion rate compared to salt or heat, so double diffusive convection is thought to be important below sediment laden rivers in lakes and the ocean.[10][11]

Two quite different types of fluid motion exist—and therefore are classified accordingly—depending on whether the stable stratification is provided by the density-affecting component with the lowest or the highest molecular diffusivity. If the stratification is provided by the component with the lower molecular diffusivity (for example in case of a stable salt-stratified ocean perturbed

by a thermal gradient due to an iceberg—a density ratio between 0 and 1), the stratification is called to be of "diffusive" type (see external link below), otherwise it is of "finger" type, occurring frequently in oceanographic studies as salt-fingers.[12] These long fingers of rising and sinking water occur when hot saline water lies over cold fresh water of a higher density. A perturbation to the surface of hot salty water results in an element of hot salty water surrounded by cold fresh water. This element loses its heat more rapidly than its salinity because the diffusion of heat is faster than of salt; this is analogous to the way in which just unstirred coffee goes cold before the sugar has diffused to the top. Because the water becomes cooler but remains salty, it becomes denser than the fluid layer beneath it. This makes the perturbation grow and causes the downward extension of a salt finger. As this finger grows, additional thermal diffusion accelerates this effect.

2.1 Governing equations

The conservation equations for vertical momentum, heat and salinity equations (under Boussinesq's approximation) have the following form for double diffusive salt fingers:[13]

$$\nabla \cdot \mathbf{U} = 0$$

$$\frac{\partial U}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = \nu \nabla^2 \mathbf{U} - g(\beta \Delta S - \alpha \Delta T) \mathbf{k}$$

$$\frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T = k_T \nabla^2 T$$

$$\frac{\partial S}{\partial t} + \mathbf{U} \cdot \nabla S = k_S \nabla^2 S$$

Where, \mathbf{U} and \mathbf{W} are velocity components in horizontal (x axis) and vertical (z axis) direction; \mathbf{k} is the unit vector in the Z -direction, k_T is molecular diffusivity of heat, k_S is molecular diffusivity of salt, α is coefficient of thermal expansion at constant pressure and salinity and β is the haline contraction coefficient at constant pressure and temperature. The above set of conservation equations governing the two-dimensional finger-convection system is non-dimensionalised using the following scaling: the depth of the total layer height H is chosen as the characteristic length, velocity (U , W), salinity (S), temperature (T) and time (t) are non-dimensionalised as[14]

$$x = \frac{X}{H}, z = \frac{Z}{H}, u = \frac{U}{k_T/H}, w = \frac{W}{k_T/H}, S^* = \frac{S - S_B}{S_T - S_B}, T^* = \frac{T - T_B}{T_T - T_B}, t^* = \frac{t}{H^2/k_T}$$

Where, (T_T , S_T) and (T_B , S_B) are the temperature and concentration of the top and bottom layers respectively. On introducing the above non-dimensional variables, the above governing equations reduce to the following form:

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t^*} + u \cdot \nabla u = Pr \nabla^2 u - \left[Pr Ra_T \left(\frac{S^*}{R_p} - T^* \right) \right] \mathbf{k}$$

$$\frac{\partial T^*}{\partial t^*} + u \cdot \nabla T^* = \nabla^2 T^*$$

$$\frac{\partial S^*}{\partial t^*} + u \cdot \nabla S^* = \frac{Pr}{Sc} \nabla^2 S^*$$

Where, R_p is the density stability ratio, Ra_T is the thermal Rayleigh number, Pr is the Prandtl number, Sc is the Schmidt number which are defined as

$$= \frac{\alpha \Delta T}{\beta \Delta S}, Ra_T = \frac{g \alpha \Delta T H^3}{\nu k_T}, Pr = \frac{\nu}{k_T}, Sc = \frac{\nu}{k_S}.$$

Figure 1(a-d) shows the evolution of salt fingers in heat-salt system for different Rayleigh numbers at a fixed R_p . It can be noticed that thin and thick fingers form at different Ra_T . Fingers flux ratio, growth rate, kinetic energy, evolution pattern, finger width etc. are found to be the function of Rayleigh numbers and R_p . Where, flux ratio is another important non-dimensional parameter. It is the ratio of heat and salinity fluxes, defined as,

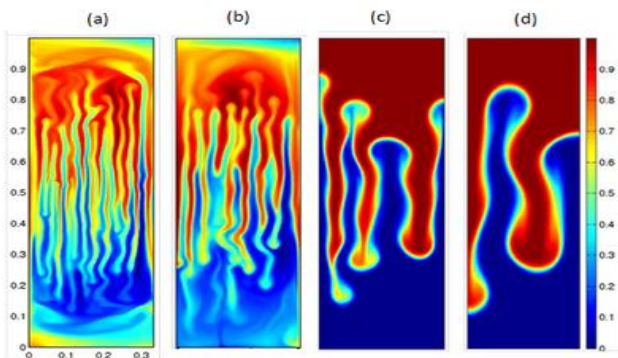


Fig. 1 Numerical simulations results show concentration fields at different Rayleigh numbers for fixed value of $R_p = 6$. [15] The parameters are: (a) $Ra_T = 7 \times 10^8$, $t = 1.12 \times 10^{-2}$, (b) $Ra_T = 3.5 \times 10^8$, $t = 1.12 \times 10^{-2}$, (c) $Ra_T = 7 \times 10^6$, $t = 1.31 \times 10^{-2}$, (d) $Ra_T = 7 \times 10^5$, $t = 3.69 \times 10^{-2}$.

It is seen from the figure that finger characteristics such as width, evolution pattern are a function of Rayleigh numbers.

3. Oldroyd-B model

The Oldroyd-B model is a constitutive model used to describe the flow of viscoelastic fluids. This model can be

regarded as an extension of the upper-convected Maxwell model and is equivalent to a fluid filled with elastic bead and spring dumbbells. The model is named after its creator James G. Oldroyd. [16]

The model can be written as:

$$\mathbf{T} + \lambda_1 \overset{\nabla}{\mathbf{T}} = 2\eta_0 (\mathbf{D} + \lambda_2 \overset{\nabla}{\mathbf{D}})$$

where:

- \mathbf{T} is the deviatoric part of the stress tensor;
- λ_1 is the relaxation time;
- λ_2 is the retardation time = $\frac{\eta_p}{\eta_0} \lambda_1$;
- $\overset{\nabla}{\mathbf{T}}$ is the upper-convected time derivative of stress tensor:

$$\overset{\nabla}{\mathbf{T}} = \frac{\partial}{\partial t} \mathbf{T} + \mathbf{v} \cdot \nabla \mathbf{T} - ((\nabla \mathbf{v})^T \cdot \mathbf{T} + \mathbf{T} \cdot (\nabla \mathbf{v}));$$

- \mathbf{v} is the fluid velocity;

- η_0 is the total viscosity composed of solvent and polymer components, $\eta_0 = \eta_s + \eta_p$;

- \mathbf{D} is the deformation rate tensor or rate of strain tensor, $\mathbf{D} = \frac{1}{2} [\nabla \mathbf{v} + (\nabla \mathbf{v})^T]$.

The model can also be written split into polymeric (viscoelastic) part separately from the solvent part:

$$\mathbf{T} = 2\eta_s \mathbf{D} + \tau,$$

where

$$\tau + \lambda_1 \overset{\nabla}{\tau} = 2\eta_p \mathbf{D}$$

4. Review of Literature

4.1 Rayleigh - Benard Convection

An in-depth explanation of the convective flow and its development was presented by Benard (1901). The first person to give an analytical treatment of the problem was Lord Rayleigh (1916) He chose boundary conditions and equations of motion that modelled the experiment Benard did and obtained linear equations for the normal modes. He used Boussinesq approximation to do so. It was done so as to determine the conditions at which the basic state breaks down. Due to the above reasons the problem of thermal instability is called Rayleigh-Benard convection. The beginning of convection depends mainly on the difference in temperature of the boundaries and was expressed as a non-dimensional quantity, the Rayleigh number. Generalization of the Rayleigh problem to other boundary conditions was done by Jeffrey (1926). Theoretical work has given the most accurate information about the flow. However, details about the roll patterns were obtained from experimental studies. Thomson (1951) examined the modifications in the Rayleigh - Jeffreys theory pertaining to slow thermal convection induced by hydromagnetic effects in a fluid exposed to magnetic field. A critical temperature gradient must be crossed for convection to set it even in the case of an inviscid fluid. An estimate value of the critical gradient was found for artificial boundary conditions. This value was large enough to be detected through experiments. Thompson also discussed the applicability of the Jeffery method of marginal stability. Convective instability under high-frequency vibration was examined by Cisse et. al. (2004). The horizontal walls were at constant temperature while the vertical walls were thermally insulated. It was shown



that mechanical quasi-equilibrium wasn't possible when the direction of the vibrations wasn't parallel to temperature gradient. This paper also showed that high-frequency oscillations can be used to delay convection. Li and Khayat (2005) examined the influence of elasticity and inertia on convection for viscoelastic fluids with a constant viscosity. The solution was arrived at using Galerkin technique. It was found that amplitude of motion wasn't influenced much by elasticity or retardation time at higher Rayleigh numbers. Oscillatory mode of convection was preferred. Kao and Yang (2007) investigated two-dimensional Rayleigh-Benard problem using Lattice Boltzmann method. The simulations showed that the unsteady periodic flows occur at a certain Prandtl number with the corresponding Rayleigh number. The Nusselt number did not depend on the Prandtl number for certain ranges.

4.2 Double Diffusion on Oldroyd-B Fluid

Prasannakumara et al. (2021) in their study focuses on the properties of flow, heat, and mass transmission. In the fields of biology and technology, there has been a lot of interest in the use of non-Newtonian fluids. Our objective is to investigate the flow of Oldroyd-B liquid over a stretching sheet using Cattaneo-Christov double diffusion and a heat source/sink because we have such a strong interest in non-Newtonian fluids. The modelling also takes into account the relaxation chemical reaction and thermophoretic particle deposition. By selecting pertinent similarity variables, the equations that represent the suggested flow are converted to ordinary differential equations (ODEs). The Runge-Kutta-Fehlberg fourth-fifth order technique (RKF-45) and a shooting scheme are used to solve the reduced equations. In order to give readers a clear grasp of how dimensionless parameters behave on dimensionless velocity, concentration, and temperature profiles, physical descriptions are strategically planned and rationalised using graphical representations. The findings show that the fluid velocity decreases as the rotation parameter's values increase. The thermal and concentration profiles decrease as the values of the relaxation time parameters of temperature and concentration increase, respectively. The thermal profile is advanced by higher values of the heat source/sink parameter. The concentration profile decreases as the thermophoretic and chemical reaction rate parameters increase in value.

Wang et al. (2019) in their study discussed about "Weak nonlinear analysis of Darcy-Brinkman convection in Oldroyd-B fluid" On the basis of the Darcy-Brinkman model, an investigation of Oldroyd-B fluid chaotic convection in a porous medium under temperature modulation is described. To deal with the stability of convection with variable Darcy numbers and Darcy-

Rayleigh numbers, a four-dimensional nonlinear system arising from a truncated Galerkin expansion of the conservation and constitutive equations is set up. The Darcy-Rayleigh number governs chaotic behaviour, whereas the Darcy number significantly affects system stability. The system will transition from steady convection to periodic or chaotic motion when the lower boundary is heated in a time-periodic way.

Vanishree et al. (2018) in their study discussed about analysis of the impact of rotation modulation in Oldroyd-B liquids under double diffusive convection. There has been both linear and non-linear analysis. The thermal Rayleigh number has been calculated using a standard perturbation method. The findings demonstrate that strain retardation parameter and Lewis number stabilise the system whereas stress relaxation destabilises it. A set of Lorentz equations is produced by the truncated Fourier series expansion, which aids in the non-linear analysis. Heat and mass transport are measured using mean Nusselt and Sherwood values, respectively. Lewis number, strain retardation parameter, and stress relaxation parameter are shown to minimise heat and mass transmission while increasing them. Modulation is observed to result in sub-critical motion.

4.3 Double Diffusive Convection

The present study of doubly diffusive convection owes its beginning to Stem (1960). This paper showed how two diffusing agents with different rates can induce convection. He noted that the stratification of salt and heat in oceans is due to the fact that the molecular diffusivity of temperature is more than that of salt diffusivity to a large extent. The convective motion in laminar regime and its stability characteristic too was covered in this.

Nithyadevi and Yang (2009) investigated two-component convection with Soret and Dufour effect. The two-component convection of water in an enclosure heated partially was considered. The effects of various parameters were depicted graphically along with those of Nusselt and Sherwood numbers.

Pranesh and Arun (2012) examined the non-uniform concentration gradient and its effects on doubly diffusive convection in micropolar fluids. The solute and heat were induced from the bottom layer. The Rayleigh number was obtained using Galerkin method for different boundary conditions. They considered a linear profile and five nonlinear profiles and found that in comparison to a fluid without any suspension in it, fluids with suspended particles that is heated from below with the addition of salt from below is more stable.

Radko (2013), in his book, spoke about the different aspects of double diffusive convection. The book ranges from the history of this type of convection, how it was



discovered, how this convection leads to the formation of salt fingers. It further gives insight into the linear stability problem based on two component convection, weakly non-linear models, The two-layer system, condition for instability, horizontal layer flow and diffusive layer among many other topics.

4.4 Temperature Modulation

The solution method for the problem pertaining to convection in the presence of temperature modulation was first introduced by Venezian (1969). The thermal perturbations on either wall were considered to be time-dependent sinusoidal. The disturbances were infinitesimally small. The perturbation was expanded in powers of amplitude of the modulation. The Rayleigh number was obtained in terms of frequency of modulation. This approach to the solution was adopted by researchers thereafter.

Rosenblat and Tanaka (1971) were one of the few early researchers to take on thermal modulation. Using a linear analysis of a fluid confined in a classical Benard geometry with the temperatures of the boundary walls being modulated, they determined the behavior of system. The effects of the oscillations on the stability were inspected. They found an enhancement in the values of critical Rayleigh number.

Davis (1976) investigated the stability of time periodic flows wherein three problems with sinusoidal time variation were reviewed. Using parallel shear flows, convective instabilities and centrifugal instabilities, scale analysis was discussed in detail. Stability of a system was described in depth.

Roppo (1984) investigated beard convection occurring due to a time- periodic heating.

This heating was done at the lower boundary. Modulation produced an array of stable hexagons near the critical Rayleigh number. This range was of the fourth order of amplitude and it decreased as when the modulation decreased.

Ahlers et. al. (1984) did an experimental investigation of convection in a layer of liquid with heat current modulation. The results were described using a theoretical generalization of the Lorenz model. The results obtained via experiments and theory matched well.

A nonlinear investigation of thermal convection in a layer of viscoelastic fluids was done by Rosenblat (1986). The solution of the study depended on the relation that is used to describe viscoelasticity. A Fourier representation of the solution was developed that exhibited aperiodic, or chaotic solutions in a particular truncation.

Meyer et. al. (1988) examined the pattern competition in thermally modulated convection. This was done using shadowgraph flow visualization. Hexagonal patterns were

observed for a range above the convective threshold. A coexistence between hexagons and rolls were observed which was followed by roll-like patterns.

The effects of amplitude on convection due to time-periodic heating was done by Antohe and Lage (1996). The natural convection activity within the enclosure was found to peak at several frequencies. The effects of the parameters were observed experimentally and comparisons with theoretical results were favourable.

Rayleigh-Benard convection with spatially periodic modulations was considered by Schmitz and Zimmermann (1996). A two-dimensional convection between two rigid walls was considered. The top container boundary was undulated, whereas the upper and lower boundaries were modulated. This thermal modulation was sinusoidal. The drift that arose in the fluid layer depended on the sign of the relative phase between modulations. The authors have described various experiments based on these.

The effects of gravity and temperature modulation in weak electrically conducting fluids in the presence of angular momentum was examined by Siddheshwar and Pranesh (1999). They found that sub-critical motion occurs due to temperature modulation. Also, modulation of gravity lead to a delay in the onset of convection. Asymptotic analysis was done for small and large- scale frequencies.

Rudraiah and Siddheshwar (2000) investigated non-uniform temperature gradient in a fluid with suspended particles. The convection was driven by surface tension. The eigenvalue was obtained using the Rayleigh- Ritz technique. The study included six different temperature profiles and a comparison between them. This showed that the fluid heated from below with suspended particles in them is more stable as opposed to Newtonian fluids.

Or and Kelly (2002) considered the case of convection induced by thermal modulation of a thin layer of fluid with deformable free surface. A linear analysis was done to find the thermo capillary instability of the system. Moderate amplitude of the modulation was found to stabilize the system. Large amplitudes, however, had the opposite effect. The effects of modulation on the temperature and heat flux were also discussed.

Chung (2004) performed a linear analysis of a second-grade fluid layer whose boundary temperatures are modulated. Rayleigh number was obtained for three different types of modulation. The start of convection could be altered when Prandtl number, modulation frequency and the viscoelastic parameter were varied.

Bhadauria (2006) studied time-periodic heating of a fluid with magnetic field. The temperature gradient had both a steady and an oscillatory part. The temperatures of the boundary walls were modulated. Floquet theory was used to arrive at the results. This study proved the stabilizing nature of magnetic field and the modulation, depending on its frequency.

Singh and Bajaj (2008) examined the stability of an



infinite fluid layer excited due to a time-periodic oscillation of the temperatures. The instability zones of amplitude and wave number were obtained. The authors noted that the instability being harmonic or sub-harmonic depended on the modulation parameters.

The impact of magnetic field in Boussinesq-Stokes suspension was investigated by Pranesh and Sangeetha (2010). The boundary wall temperatures were modulated with respect to time. They considered synchronous, asynchronous and lower wall modulations. Sub-critical motion was observed, and modulation could be controlled in order to get a desirable behaviour from the system.

Bhadauria et. al. (2012) made a nonlinear analysis of temperature modulation and g-jitter in viscous fluid layer with rotation. The Ginzburg-Landau equation was used to analyse the effects of modulations. Rotation can be used in modify the behaviour of the system.

Siddheshwar et. al. (2016) did an analysis of doubly diffusive convection. The couple stress liquid considered was thermally modulated. The transfer of heat and mass were calculated using Sherwood and Nusselt numbers that are dependent on time. It was shown that increasing the values of the Prandtl number, solutal Rayleigh number and Lewis number increased the transfers. On the other hand, the increased values of couple stress parameter decreased them. These results were true in all three cases of modulation.

4.5 Convection under Gravity Modulation

Study of convection under gravity modulation has been studied widely. Their applications make this a very interesting topic. Benjamin and Ursell (1954) investigated gravity modulation on standing waves of a liquid contained in a vessel. This was concentrated on Mathieu's equation. The linear analysis done was used to find the impact of modulation when the heating was done both from above and below.

The study of modulation of gravity in a layer of liquid was initially done by Gershuni and Zhekhovitskii (1963). The equilibrium of a layer of fluid maintained horizontally was investigated. The solution of this system puts forth the characteristic singularity of the problem.

Another one of the early studies on gravity modulation was done by Gresho and Sani (1970) who considered the instability of a fluid layer that was heated from below subjected to gravity modulation. The time dependent body force was generated by vibrating the fluid layer. The approximate solution gave an insight into the qualitative results which gave a more complete analysis of the stability. The results were extended to the case of simple pendulum. Few effects of the finite amplitude flows were discussed as well. In the study of impact of g-jitter on transfer of heat, Amin (1988) found that small fluctuations

in the gravity produced a body force that was fluctuating. This resulted in a flow of the fluid that had both a time-independent and time-dependent component. Biringen and Peltier (1990) did a numerical simulation of 3D convection subjected to gravitation modulation. A semi-implicit, pseudo-spectral method was employed to solve the nonlinear Boussinesq Navier-Stokes equation.

Thermosolutal convection in a fluid layer under g-jitter was considered by Saunders et. al. (1992). Fingering and diffusive regimes of thermosolutal convection were mainly considered. It was found that modulation may either stabilizes or destabilizes the base solution. The instability arose due to a complex conjugate mode as well. Regions of instability occurred, and it exhibited a coupling with the unmodulated frequency of oscillation. Under high frequency of modulation, subcritical motions took place.

Two-dimensional convection under gravity modulation was examined by Clever et. al. (1993). The oscillatory convection was studied using a Galerkin method. Nonlinear solutions of non-dimensional frequencies and amplitude were obtained. It was found that synchronous convection with finite amplitude is unstable to the subharmonic modes.

Christov and Homsy (2001) performed nonlinear analysis of twodimensional convection with gravity modulation. The gravitational acceleration consisted of a mean and a harmonic modulation. The numerical solution was calculated using an economic operator splitting scheme along with internal iterations within given step time. The fluid inertia was negligible as the Prandtl number was taken to be high. Only thermal inertia was responsible for the convection.

Deka and Soundalgekar (2006) analysed the impact of g-jitter on convection in a fluid flowing through a vertical isothermal plate. An exact solution to this problem was derived using Laplace transform method. So are the penetration distance, velocity profiles and the skin-friction. When the frequency of modulation and the Prandtl number were increased there was a decrease in the transient velocity. With increase in the values of omega, transition from conduction to convective form of heat transfer was delayed.

Yu et. al. (2007) carried out a linear analysis of two-component convection with gravity modulation. The subsequent nonlinear analysis was done too. The solute gradient decreased slowly to make the layer unstable as the boundaries were non-diffusive. The theory is complimented with the corresponding experiment designed in such a way that no internal wave stability mode was excited. It showed that increasing the values of solute Rayleigh number caused the modulation to destabilize the system.

Study of convection in a micropolar fluid under gravity modulation was done by Pranesh et. al. (2014). A perturbation technique was employed for the linear



analysis, whereas the nonlinear analysis was done with Fourier series representation of the modes. The Lorenz model was numerically solved to find the transfer of heat. The effects of the various parameters were discussed.

Kiran and Narasimhalu (2017) performed a weakly nonlinear analysis of oscillatory convection in a liquid exposed to gravity modulation. The fluid was taken to be electrically conducting. The solvability condition convection yielded an amplitude equation. They found that the Prandtl number, amplitude of modulation and magnetic Prandtl number increased the amount of heat transport. Modulation frequency and Chandrashekar number reduced the heat transfer.

5. Conclusion

Investigating the start of convection in a horizontal layer of modified Darcy porous media saturated with an Oldroyd-B viscoelastic nanofluid requires taking into account the physically plausible boundary condition that the flux of nanoparticle volume fraction is zero on the boundaries. The Galerkin method is used to quantitatively solve the generalised eigenvalue problem. The study assists in determining the consequences of externally modulating the convection. They either speed up or slow down convection. The following judgements are reached:

1. The different parameters in the case of in-phase modulation lead to a delay in convection.
2. For in-phase and out-of-phase modulations, the parameters have opposite effects.
3. Out-of-phase modulations produce the same outcomes as lower wall modulations.
4. Modulation is a highly effective method of controlling convection.
5. For in-phase modulation, Lewis number and strain retardation reduce the heat/mass transmission.

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